33B midterm 1

Vedant Sahu

TOTAL POINTS

38 / 40

QUESTION 1

integration factor 8 pts

1.1 integration factor 4 / 4

✓ **- 0 pts** **Correct**

- 1 pts minor mistake

- 4 pts no work

- 3 pts subtle work, try to find h(x) but equation incorrect

- 2 pts get h(x),but not u(x)

- 2 pts get u(x) but without details; know how to get u(x) but calculate incorrectly

1.2 solve 4 / 4

✓ **- 0 pts** **Correct**

- 1 pts solution should be in form of F(x,y) = c

- 4 pts no work

- 3 pts know need to do partial integration, but incorrect.

- 2 pts correct form F = \phi + xxxx, but \phi incorrect ; or the other way around.

- 1 pts minor mistake

QUESTION 2

separable eon 12 pts

2.1 explicit solution 5 / 5

✓ **+ 1 pts Separating the Equation**

✓ **+ 1 pts Partial Fractions**

✓ **+ 1 pts Computing Integral**

✓ **+ 1 pts Log Rule Application**

✓ **+ 1 pts Computing Solution**

+ 2 pts Bernoulli Transformation

+ 1 pts Integrating Factor

+ 2 pts Rest of Bernoulli Solution

+ 0 pts No points

Technically correct, but the arbitrariness of C means you can drop the absolute value.

2.2 y(1) = 2 2 / 2

✓ **+ 2 pts Correct Answer**

+ 1.5 pts Correct Answer, Wrong Solution

+ 1 pts Knowing the Process

+ 0 pts No points

2.3 interval of existence 1 / 3

+ 1 pts Knowing 0 is not included

+ 1 pts Correct for their function

+ 1 pts Correct

✓ **+ 1 pts Knowing 2 is not included.**

+ 0 pts No points

2.4 y(1) = 0 2 / 2

✓ **+ 2 pts Correct Answer**

+ 1 pts Correct Answer, but on accident

+ 0 pts No points

QUESTION 3

3 mixing problem 7 / 7

- 1 pts Identifying x'= rate in- rate out, rate in = 4

- 2 pts Identify rate out = x/(50+t)

- 1 pts Find an integrating factor or homogeneous solution

- 2 pts Find the general solution

- 1 pts Incorporate the initial condition.

✓ **- 0 pts Correct**

- 1 pts Accidentally made equation Homogeneous/ too simple.

- 1 pts Forgot a factor of 2 in rate out.

QUESTION 4

exact eqn 7 pts

4.1 not exact 3 / 3

✓ **- 0 pts Correct**

- 3 pts No answer

- **- 2 pts** wrong derivatives
- **- 1 pts** wrong Q derivative
- **- 3 pts** wrong approach
- **- 1 pts** why?
- **- 1 pts** wrong P derivative

4.2 integration factor **4 / 4**

- ✓ **- 0 pts** Correct
- **- 1 pts** sign mistake
- **- 3 pts** only formula
- **- 1 pts** a=? b=?
- **- 4 pts** wrong/no work
- **- 2 pts** right start

QUESTION 5

SA 6 pts

5.1 dir field **4 / 4**

- **- 2 pts** No 2. solution
- **- 2 pts** No 1. solution
- **- 1 pts** mistake 1. solution
- **- 1 pts** mistake 2. solution
- **- 4 pts** doesn't go through the right points
- **- 2 pts** doesn't go through the right point 1. solution
- ✓ **+ 4 pts** correct

5.2 Y/N **2 / 2**

- **- 0.5 pts** 1 incorrect
- **- 1 pts** 2 incorrect
- **- 1.5 pts** 3 incorrect
- **- 2 pts** all incorrect
- ✓ **+ 2 pts** correct

MIDTERM 1

10/24/2018

Math33B
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Problem	Points	Score
1	8	
2	12	
3	7	
4	7	
SA	6	
Total	40	

Exercise 1. (8pt)
Consider the differential equations

$$2y^2 + 4x^2 + 2xy\frac{dy}{dx} = 0$$

(1) Find the integrating factor for the above equations.(4pt)
(Hint: it only depends on x)

$$h(x) = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$$

$$= \frac{1}{2xy}(4y - 2y) = \frac{2y}{2xy}$$

$$= \frac{1}{x}$$

$$\mu(x) = e^{\int h(x)\,dx} = e^{\int \frac{1}{x}\,dx}$$

$$= e^{\ln|x|}$$

$$= |x|$$

(2) Solve the equation.(4pt)

$$|x|(2y^2 + 4x^2)\,dx + 2|x|xy\,dy = 0$$

$$F(x, y) = \int 2x^2y\,dy + \phi(x)$$

$$= x^2y^2 + \phi(x)$$

$$\frac{\partial F}{\partial x} = 2xy^2 + \phi'(x) = 2xy^2 + 4x^3$$

$$\Rightarrow \phi'(x) = 4x^3$$

$$\Rightarrow \phi(x) = \int 4x^3\,dx = x^4$$

$$F(x,y) = x^2y^2 + x^4$$

$\Rightarrow x^2y^2 + x^4 = C$ is the solution.

The solution will be the same for $x > 0$ and $x < 0$

Exercise 2. (12pt) Consider the differential equation

$$\frac{dy}{dx} = \frac{y^2 - y}{x}$$

(1) Find the explicit general solution. (5pt)

$$\frac{dy}{y(y-1)} = \frac{dx}{x}$$

$$\frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}$$

$$A(y-1) + By = 1$$

$y = 0$: $-A = 1 \Rightarrow A = -1$

$y = 1$: $B = 1$

$$\int \left(-\frac{1}{y} + \frac{1}{y-1} \right) dy = \int \frac{dx}{x}$$

$$-\ln|y| + \ln|y-1| = \ln|x| + c$$

$$\ln\left|\frac{y-1}{y}\right| = \ln|x| + c$$

$$|1 - 1/y| = \sout{|x|}\ e^{\ln|x| + c} = e^c |x|$$

(2) Find the solution to this equation that satisfies the initial condition $y(1) = 2$. (2pt)

(1) Continued

$1 - 1/y = A|x| \quad A \in \mathbb{R}$

$\Rightarrow 1/y = 1 - A|x|$

$\Rightarrow y = \dfrac{1}{1 - A|x|}$

(2) $y(1) = 2$

$\Rightarrow 2 = \dfrac{1}{1-A} \Rightarrow 1 - A = \dfrac{1}{2} \Rightarrow A = \dfrac{1}{2}$

Therefore, $y = \dfrac{1}{1 - 1/2|x|} = \dfrac{2}{2 - |x|}$

(3) What is the interval of existence of the solution you found in (b). (3pt)

The interval should contain $x = 1$

$2 - |x| < 0$ or $2 - |x| > 0$

$\Rightarrow |x| > 2$ or $|x| < 2$

Doesn't contain $x = 1$ — contains $x = 1$

$|x| < 2$

$\Rightarrow x \in (-2, 2)$

Therefore, the interval of existence is $(-2, 2)$

(4) Find the solution to this equation that satisfies the initial condition $y(1) = 0$. (2pt)

$$y(x) = \frac{1}{1 - A|x|}$$

This cannot be 0 for any value of A

The only solution that satisfies the initial condition $y(1) = 0$. is

$y(x) = 0$

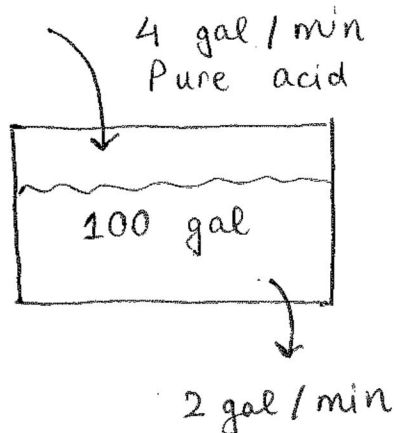
$\frac{dy}{y}$ $\frac{dy}{dx} = 0 = \frac{y^2 - y}{x}$

Hence, $y(x) = 0$ is the required solution

Exercise 3. (7pt) Suppose there is a tank filled with 100 gallons of water. Pure acid flows into the tank at a rate of 4 gal/min and the well mixed solution leaves the tank at the of 2 gal/min rate. Let $x(t)$ be the volume in gallons of acid in the tank at time t. Find $x(t)$ for any given time t.

4 gal / min
Pure acid

100 gal

2 gal / min

$x(t)$: Volume of acid in the tank at time t (in gallons)

$V(t) = 100 + 2t$

$x' = \text{rate in} - \text{rate out}$

$= 4 - \frac{x}{100+2t} \cdot 2 = 4 - \frac{x}{50+t}$

$x' + \frac{x}{50+t} = 4$

$u(t) = e^{\int \frac{dt}{50+t}} = e^{\ln|50+t|} = 50+t$

$(t > 0,$ so $|50+t| = 50+t)$

$(50+t)x' + x = 4(50+t)$

$((50+t)x)' = 200 + 4t$

$(50+t)x = \int (200+4t)\,dt = 200t + 2t^2 + C$

$x(t) = \frac{200t + 2t^2 + C}{50+t}$

$x(0) = 0 \Rightarrow C = 0$

Therefore, $x(t) = \frac{200t + 2t^2}{50+t}$

Exercise 4. (7pt) Consider

$$4yxdx + 5x^2dy$$

(1) Show that the above equation is not exact. (3pt)

$\frac{\partial P}{\partial y} = 4x$ $\frac{\partial Q}{\partial x} = 10x$

$\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$

Therefore, the above equation is not exact.

(2) Find a and b such that $x^a y^b$ is an integration factor of the above equation. (4pt)

$h(x) = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$

$= \frac{1}{5x^2}(4x - 10x) = \frac{-6}{5x}$

$u(x) = e^{\int h(x)dx} = e^{\int \frac{-6}{5x}dx}$

$= e^{-6/5 \ln|x|} = |x|^{-6/5}$

$u(x)P = 4yx^{-1/5}$

$u(x)Q = 5x^{4/5}$

$\frac{\partial}{\partial y}(4yx^{-1/5}) = 4x^{-1/5}$

$\frac{\partial}{\partial x}(5x^{4/5}) = 4x^{-1/5}$ } exact

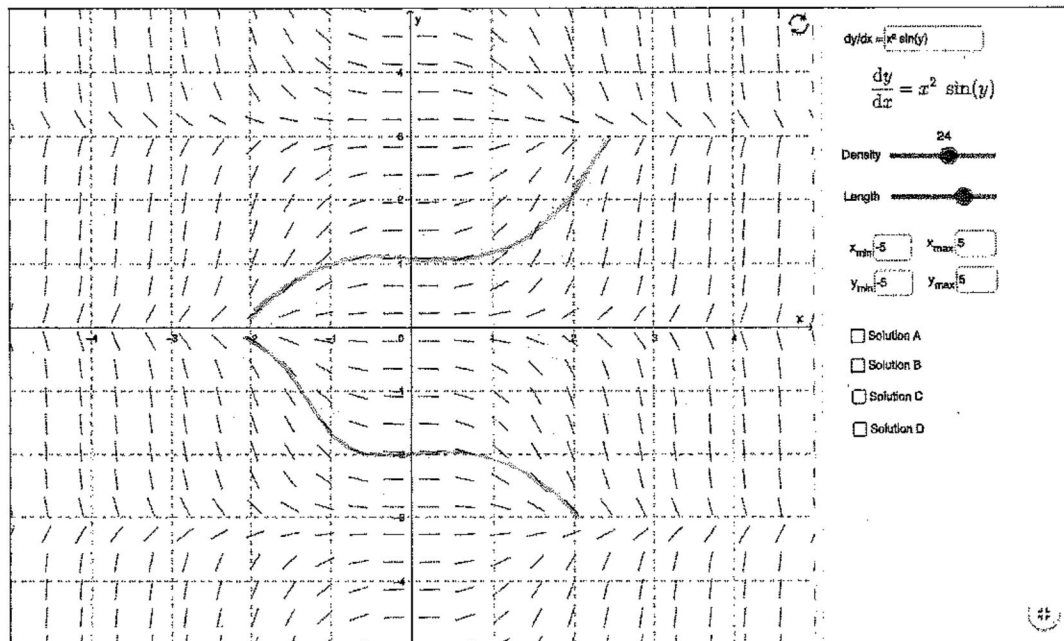
$x^a y^b = x^{-6/5} y^0$ $a = -6/5$, $b = 0$

Taking $u(x) = 1/x^2y$ will also work

$4/x\, dx + 5/y\, dy$

$\frac{\partial}{\partial y}\left(\frac{4}{x}\right) = \frac{\partial}{\partial x}\left(\frac{5}{y}\right) = 0$

~~a = 2~~, ~~b = 1~~ $a = -2$, $b = -1$

Field M1 F18.png

1. Short answer problems

(no explanation needed)

(1) (4pt) Consider the above direction field and draw the solution through (0,1) and the solution through (0,-2).

(2) (2pt) Which of the following are homogeneous differential equations?

(Y) / N $\sin(\frac{x}{y})dy + 2dx = 0$ Yes

Y / (N) $(xy + x^2)dy + (y^2x - x^2y)dx$ No

Y / (N) $\sin(xy)dy - \cos(xy)dx$ No

(Y) / N $\sqrt{x^2y^2 - 4xy^3}dy + x^2dx$ Yes